Modelling the dustospheres of AGB stars

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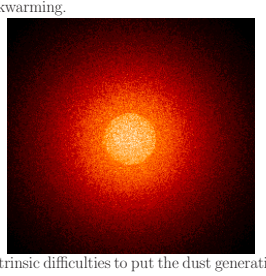
Introduction

Dust formation is a common phenomenon in stellar environments. The formation of dust takes place very close or even inside the stellar atmosphere, triggered by levitating mechanisms. The transition region between the stellar atmosphere and the circumstellar envelope of AGB stars is not very well-defined and strongly influenced by the transport properties of newly formed dust grains, e.g. via radiation pressure or backwarming.



The intrinsic difficulties to put the dust generating region into the common nomenclature of stellar structure lead us to call this dynamic region the *dustosphere* (greek: *Kornioktosphere*) of the star. Although we restrict ourselves in the following to the dustospheres of AGB stars, we expect this definition to be useful for a variety of other objects, too.

The poster presents a reliable code which has been designed for the numerical simulation of pulsationally levitated atmospheres of AGB stars. The CHILD-code has initially been developed during the early 1990's by Fleischer, Gauger and Sedlmayr (1992) and since then served as a platform for different studies and additional implementations.

The Physical Situation

The complex interplay of the different physical processes that determine the structure of the dustosphere is sketched in Fig. 1.

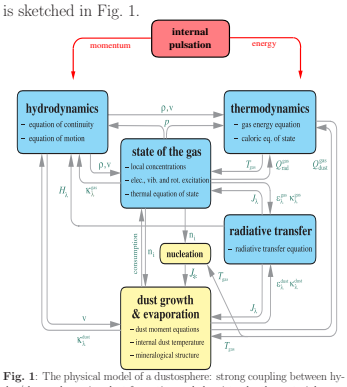


Fig. 1: The physical model of a dustosphere: strong coupling between hydro/thermodynamics, dust formation and chemistry by the material properties of dust and gas. Stellar interior pulsation triggers the dust formation.

The stellar atmosphere is levitated by the combined effect of pulsationally generated acoustic waves steepening up to shocks and radiation pressure on molecules. Hence, the gas density around the star is considerably increased and thermodynamic conditions favorable to efficient formation and subsequent growth of dust particles are established. The dust is strongly coupled to the stellar radiation field by its large opacity, $\kappa^{\rm dust}$. It is accelerated outwards by radiation pressure, while the gas is dragged outwards by frictional coupling to the dust: a *dust driven wind* can establish.

The material inside of a newly formed dust layer is heated via *backwarming* from the optically thick dust shell which causes the dust nucleation to stop, until the shell has moved away. This *feedback* introduces a new time scale into the dynamical system which is determined by the interplay of dust formation and growth and its influence on the hydrodynamics (*multiperiodicity*, Fleischer et al. 1995, Winters et al. 1994). An onion-like *shell structure* results.

The strong coupling of these different physical processes (arrows in Fig. 1) requires a *simultaneous treatment* of the macroscopical domain including hydrodynamics and radiative transfer as well as the microphysics including chemistry, dust nucleation and growth, state-, and material functions. The microphysics determines the local strength of the coupling (e.g. Woitke et al. 1996, Schirrmacher 2000, Helling et al. 2000).

The Modeling Approach

The dustosphere is assumed to be spherically symmetric which reduces the macroscopic modeling to one dimension. Since AGB stars do not generally show features that would explicitly require a multi-dimensional treatment - like disks or jets - this assumption does not hide any first order effects. On the other hand, this simplification gives the opportunity to implement more elaborate treatments of the microphysical processes.

Basic Equations

Adopting the Lagrangian formalism of a co-moving frame, the model approach follows that of a rarefied compressible fluid (all quantities have their usual meaning):

- **Equation of motion:**

$$\frac{Du}{Dt} = -V_0\left(\frac{r}{r_0}\right)^2\frac{\partial p}{\partial r} + \mathcal{F}$$
$$\mathcal{F} = -\frac{GM(r)}{r^2}(1-\alpha) \qquad \alpha = \kappa\frac{L_*}{4\pi\, c\, G\, M(r)}$$

V_0 - volume of a mass element at $t = 0$, r_0 - position of the mass element at $t = 0$, $M(r)$ - total mass at radial position r, α - total radiative acceleration in units of the gravitational deceleration g, $\kappa = \kappa^{\rm dust} + \kappa^{\rm gas}$ - total opacity (see. Sect. 4).

- **Equation of continuity:**

$$\frac{D\rho}{Dt} + \boldsymbol{u}\,\boldsymbol{\nabla}\rho = 0 \quad \Rightarrow \quad \frac{Dr}{Dt} = u$$

- **Energy equation:**

$$\frac{D\,e(T_G,\rho,...)}{D\,t} = -pdV + \dot{Q}_{\rm rad} + \dot{Q}_{\rm vis}$$

$-pdV$ hydrodynamic energy input, $\dot{Q}_{\rm rad}$ - radiative heating/cooling rate, $\dot{Q}_{\rm vis}$ - heating by the artificial viscosity (see *Numerics*).

- The **radiation transfer** problem is treated by the moment method from Unno & Kondo (1976) for a spherically symmetric, time-independent, grey LTE atmosphere in radiative equilibrium:

$$\mu\frac{\partial I}{\partial r} + \frac{1-\mu^2}{r}\frac{\partial I}{\partial \mu} = -\kappa\,\rho(I-J)$$
$$\frac{d\mu(r)}{dr} = \frac{1-\mu(r)^2}{r\mu(r)} - \frac{2}{3}\kappa\rho$$

$\mu = \cos\theta$ - separation angle, I - specific intensity and J mean intensity of the radiation field.
In order to calculate the dust opacities $\kappa^{\rm dust}(\rho, T_G)$ the *dust moment equations* (Gail & Sedlmayr 1988, Gauger et al. 1990) have to be solved:

- **Dust moment equations:**

$$\frac{D\,K_0}{D\,t} = \frac{J_*}{n_{<\rm H>}}$$
$$\frac{D\,K_{\rm j}}{D\,t} = N_1^{\rm j/3}\frac{J_*}{n_{<\rm H>}} + \frac{j}{3\tau_{\rm gr}}K_{\rm j-1}$$

$n_{<\rm H>}K_0 = n_{\rm d}$ - dust number density, $n_{<\rm H>}K_3 \sim$ total dust volume, J_* - net nucleation rate, $\tau_{\rm gr}$ - growth time scale, j=1,2,3 - order of the dust moments.

- The **chemical abundances** are calculated in chemical equilibrium applying the law of mass action complemented by the equation for element conservation.

Initial and Boundary Conditions

The system of basic equations is completed by a set of initial and boundary conditions:

- The dynamical calculations are started from a **hydrostatic dust free model**.
- The stellar pulsation is simulated by a sinusoidally varying velocity at the inner boundary with prescribed period P and piston amplitude Δu (**piston approximation**).
- The optical depth τ is forced to vanish at the outer boundary: $\tau(r_{\rm max}) = 0$.
- At the inner boundary the flux is constant and the separation angle is zero: $\mu(r = R_{\rm inner}) = 0$.
- The consistency condition in the radiative transfer is choosen as: $T_{\rm RE}(r = R_\star) = T_{\rm eff}$.

Material Functions

Finally, we have to provide a set of material functions in order to close the system of equations.

- The thermodynamic **state functions** give the dependence of pressure and internal energy on the thermodynamic state variables:

$$p = p(T_G, \rho, ...)$$
$$e = e(T_G, \rho, ...).$$

- The **heating/cooling function** describes the energy exchange of the gas by radiative processes:

$$\dot{Q} = \dot{Q}(T_G, J_\nu, \rho, ...).$$

- The **net nucleation rate**, $J_* = J_{\rm nuc}(c_i(T_G,\rho), T_G) - J_{\rm evap}(T_{\rm dust}, ...)$, describes the formation and destruction of dust.
- The **net growth rate** describes the growth and the thermal evaporation of dust particles:

$$\tau_{\rm dust} = \tau_{\rm dust}\left(c_i(T_G,\rho), T_G, T_{\rm dust}...\right).$$

In the grey approximation $T_{\rm dust} = T_{\rm RE}$ is applied.

- The **opacity** $\kappa = \kappa^{\rm dust} + \kappa^{\rm gas}$ of the gas/dust mixture describes the coupling of the circumstellar material to the stellar radiation field:

$$\kappa^{\rm dust} = \kappa^{\rm dust}(K_3, ...)$$
$$\kappa^{\rm gas} = \kappa^{\rm gas}\left(c_{\rm i}(T_G,\rho), T_G, ...\right).$$

Numerics

The equation of motion is discretized according to the Langrangian scheme of Richtmyer & Morton (1967) and solved by explicit forward integration in time and space. The time step width is adapted dynamically and zones crossing an arbitrary outer boundary are cut off, while new zones are carefully introduced in regions with high gradients. Each time step starts with the calculation of the new hydrodynamic structure. Shock waves are broadened using the artificial tensor viscosity of Tscharnuter & Winkler (1979). The gas temperature is determined by iterating the energy equation, new pressure and molecular abundances are calculated. The radiative transfer calculation gives the new radiative equilibrium temperature structure and finally the formation, destruction and growth of the dust particles are determined.

Implementation of Detailed Microphysics

Non-LTE Cooling & State Functions

Schirrmacher (2000) has investigated the influence of non-LTE cooling and state functions on AGB dustospheres. The data have been provided by Woitke (1997) including non-LTE effects by calculating all relevant electron, atom, ion and molecule concentrations, while the level populations rates are calculated in statistical equilibrium according to the backgroun continuum radiation field. The non-LTE effects by state and cooling functions depend on two additional parameters: the external radiation field J_ν and a locally averaged mean velocity gradient $\left\langle\frac{dv}{dr}\right\rangle$, required to describe lines using the Sobolev approximation.

For application in the CHILD-code, the stellar radiation field J_ν is assumed as approximated by a Planckian $B_\nu(T_{\rm RE})$, i.e. the vector J_ν is substituted by a single parameter, the radiative equilibrium temperature $T_{\rm RE}$.

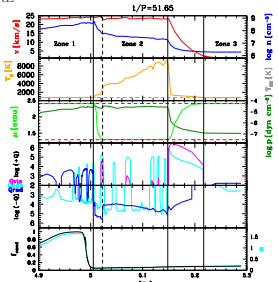


Fig. 2: Two-step-cooling behind a shock-front (in Zone 2, divided by the vertical black dashed line). all quantities have the usual meaning as in the text except: μ (Panel 3) - mean molecular weight in units of the proton mass, $Q_{\rm sd}$ (Panel 4) corresponds to the hydrodynamical energy input and $f_{\rm cond}$ (Panel 6) is the degree of condensation.
Model Parameters: M_* =1.2 $M_\odot$, T_* = 2600K, L_* = 5860 $L_\odot$, C/O=1.8, $\Delta u = 2{\rm km\ s^{-1}}$, P = 650d.

The models reveal that the cooling of the gas heated by energy dissipation of an outwards moving shock front takes place in a two step process (see Fig. 2): the first phase is characterized by the rather inefficient cooling of hydrogen atoms, while the second is dominated by the interplay of efficient cooling of polar CO-molecules and a significantly increased heat capacity due to the potential energy of newly formed H_2-molecules. The radiative relaxation zone can thereby reach the order of one stellar radius.

Influence of Gas Opacities

Temperature and density dependent mean gas opacities have been calculated adopting extensive molecular data from the Copenhagen data base in addition to several continuum sources (Helling et al. 2000). This, for the first time, allows to study the influence of the metal abundances on AGB dustospheres.

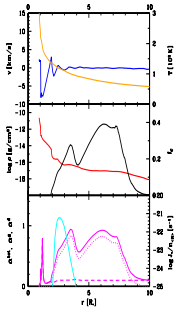


Fig. 3: Hydrodynamic model of an LMC AGB dustosphere at $t = 176\,P$. **Model parameters**: $T_* = 2600$K, $L_* = 10^4 L_\odot$, $M_* = 1M_\odot$, C/O=1.8, $\Delta u = 2{\rm km\ s^{-1}}$, $P = 650$d. blue - velocity u [km s^{-1}], orange - temperature [K]; red - gas density $\log\rho$ [g cm^{-3}], black - degree of condensation f_c; magenta: solid - total acceleration ratio $\alpha^{\rm tot}$, dotted - $\alpha^{\rm d}$, dashed - $\alpha^{\rm g}$, cyan nucleation rate $\log J_*$.

Molecules have a direct influence on the density structure of the atmosphere via radiative transfer effects — especially in the dust free regions. They further can exert a considerable force on the gas (Fig. 3, lower panel, $r \approx 1R_*$) by radiation pressure resulting e.g. in local pressure inversions, especially in models for solar metallicity stars.

In the metal-deficient case depicted in Fig. 3, the amount of strong molecular opacity carrier is smaller than for solar abundances and less material is available for dust formation. Consequently, the optical depth of the dustosphere is not large enough to exert a considerable backwarming which is responsible for the onion-like structure in AGB dustospheres.

Examplary Applications

Dynamical models of dustospheres have been applied to study e.g.

- wind properties like mass loss rate and final outflow velocity (e.g. Winters et al. 2000a)
- CO-line profiles, theroretical lightcurves, spectral energy distributions (Winters et al. 1993, 2000b)
- non-equilibrium chemistry (Patzer et al. 1999)
- warm molecular layers (Woitke et al. 1999)
- a mass loss formula in the frame work of evolution modelling resulting in a super wind (Arndt et al. 1997, Schröder et al. 1999)
- oxygen-rich circumstellar envelopes (Jeong 2000)

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