Dust formation, grain growth and evaporation under the influence of temperature fluctuations

V. Schirrmacher, U. Dirks
(Zentrum für Astronomie und Astrophysik, TU Berlin, D-10623 Berlin, Germany)

Correspondence to: vasco@astro.physik.tu-berlin.de, uli.dirks@tu-berlin.de

Overview

The treatment of the astrophysical dust complex (i.e. dust formation, grain growth and evaporation) yields intrinsic memory effects, originating from the fact, that the evaporation of dust depends on the size distribution of dust particles, which again depends very sensibly on the exact growth history. The presence of such memory effects makes the investigation of the influence of thermodynamical fluctuations on the dust physics particulary interesting. For example, thermodynamic conditions which are not favourable for the *formation* of new dust in the deterministic case, might not necessarily be so hostile that dust – formed during a random temperature fluctuation – will reevaporate. As a consequence, dust might be present in situations, which under a strict deterministic treatment should be dust-free.

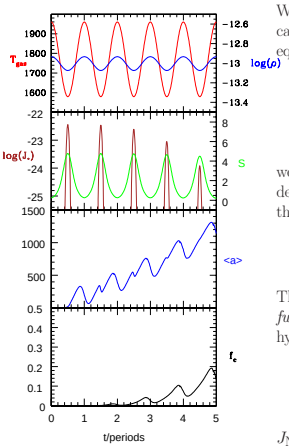

Fig. A: Dust phase memory. Gas-box simulation with a sinusoidal oszillation of $T_{\rm gas} = 1770K \pm 190K$, the density is adiabatically coupled to $T_{\rm gas}$ with $\rho_{\rm mean} = 10^{-13}$g/cm^3 and $\gamma = 5/3$. In the equilibrium position the vapour is supersaturated but below the critical supersaturation needed for the onset of nucleation. The oszillation triggers periodic nucleation events (see panel 2). Since the dust is not evaporating completely in the hot phase of the period, a stepwise growth process is initiated as is indicated by the growing mean particle size <a>(panel 3) and the increasing degree of condensation f_c(panel 4).

We investigate the impact of stochastic temperature fluctuations on dust nucleation, growth and destruction by numerical evaluation of a set of Fokker-Planck-Equations. The equations have been formulated and solved for some exemplaric situations by Dirks (2000). The results suggest that the inclusion of temperature fluctuations does indeed facilitate the formation of circumstellar dust.

The modelling method

A detailed description of this approach can be found in Dirks (2000). We follow a gas element on a deterministic trajectory of a stellar wind. In contrast to the deterministic case, we regard the local density $\rho(r)$, and temperature structure $T_{\rm gas}(r)$ of the deterministic wind as *mean values* of stochastically fluctuating quantities. In practice, such fluctuations could be realised by acoustic waves originating from the star, the dissipation of shock waves or the like.
The dust complex is treated using the dust moment method developed by Gail et al. (1984) and Gauger et al. (1990), which describes dust formation via a two step process of nucleation and subsequent grain growth. In this approach we do not account for any feedback effects from the newly formed dust to the wind, like acceleration by radiation pressure on dust, backwarming, etc. Radiative transfer calculations are not performed.
We assume fluctuations of the gas temperature given by a Gaussian distribution function $p(T)$, characterized by the parameter σ describing the *amplitude* of the fluctuation. We assume the local density to couple adiabatically to the temperature fluctuations. In contrast to the deterministic situation, where the thermodynamic conditions of the stationary wind are a function of the location r only, we are now considering a bundle of stochastic paths through an (r,T)-plane, where $T_{\rm gas}(r,T)$ and $\rho(r,T)$ are given by:

$$\begin{aligned} T_{\rm gas}(r,T) &= T_{\rm mean}(r) \pm T \\ \rho(r,T) &= \rho_{\rm mean}\left(\frac{T_{\rm gas}(r,T)}{T_{\rm mean}}\right)^{\frac{1}{\gamma-1}} \\ p(T) &= \sqrt{\frac{1}{2\pi\sigma^2}}\, e^{-\frac{T^2}{2\sigma^2}}. \end{aligned}$$

Note: T stands for the *deviation* from the mean temperature $T_{\rm mean}(r) = T_{\rm gas}(r,0)$.

$T_{\rm mean}(r) = T_{\rm gas}(r,0)$ and $\rho_{\rm mean}(r) = \rho(r,0)$ are given by the corresponding values of the deterministic wind. Note that in the case of a stationary wind the location r is strictly linked to the travelling time from the stellar surface t by $r(t) = \int_0^t v(t')dt'$.
Whereas, in the deterministic case the evolution of the dust complex can be obtained by a simple forward integration of the dust moment equations (the quantities are explained below),

$$\begin{aligned} \frac{d}{dt}K_0 &= J_{\rm N_\ell} \\ \frac{d}{dt}K_\mu &= \frac{\mu}{3}\frac{1}{\tau_{\rm net}}K_{\mu-1} + N_\ell^{\mu/3}J_{\rm N_\ell}, \quad (\mu = 1,2,3), \end{aligned} \tag{1}$$

we now have to solve a set of coupled *Fokker-Planck-Equations*, which describe the interaction of the various possible stochastical pathways through the $(r(t),T)$-plane:

$$\frac{\partial}{\partial t}q_\mu = -\frac{1}{\lambda}T\frac{\partial}{\partial T}q_\mu + \frac{\sigma^2}{\lambda}\frac{\partial^2}{\partial T^2}q_\mu + A_{\mu\nu}q_\nu + b_\mu, \ (\mu = 0,1,2,3). \tag{2}$$

The K_μ in Eq. 1 are the moments of the *particle size distribution function* $f(N,r)$, which contains the number of clusters of size N per hydrogen core at distance r from the star.

$$K_\mu(r) = \sum_{\rm N_\ell}^{\infty} N^{\frac{\mu}{3}} f(N,r)$$

$J_{\rm N_\ell}$ is the creation rate of clusters of size N_ℓ from the gas phase. $1/\tau_{\rm net}$ is the net growth rate. In Eq. (2) we have introduced the correlation time λ, which can be mulitiplied by the local wind velocity to obtain the correlation length of the fluctuations. The term $A_{\mu\nu}q_\nu + b_\mu$ corresponds to the right hand side of the deterministic system (Eq. 1) The moments K_μ are linked to the q_μ in Eq. (2) via

$$\int q_\mu(r,T)p(T)dT = E(K_\mu)(r),$$

i.e. integrating the q_μ, weighted with the probability distribution p(T), delivers the expectation values of the deterministic moments K_μ of the dust distribution function.
The coupled system of Fokker-Planck equations, is finally solved by standard numerical methods.
For each point in the (r,T)-plane the chemical equilibrium abundancies of the dust forming species (C, C_2, C_2H, C_2H_2) aswell as dust nucleation, grain growth and evaporation are calculated.

Results

As an example we show two parameter-studies for σ and λ taken from Dirks (2000). The Fokker-Plank-System (Eq. 2) has been solved for the stationary wind of a sperically symmetric C-rich Asymptotic Giant Branch (AGB) star, with a constant velocity.
The parameters of the wind are:

$$\begin{aligned} T_{mean}(r) &= T_*\sqrt{(R_*/r)} \\ \rho_{mean}(r) &= \frac{1}{4\pi r^2 v}\dot{M} \\ R_* &= 3.7\times10^{13}\text{cm} \\ T_* &= 2500\text{K} \\ L_* &= 10^4 L_\odot \\ M_* &= M_\odot \\ \dot{M} &= 2\times10^{-5}M_\odot/\text{yr} \\ v &= 20\text{km/s} \\ \epsilon_{\rm C} : \epsilon_{\rm O} &= 3 \end{aligned}$$

This simple wind model puts the emphasis on the effects introduced by the temperature fluctuations. Fig. B shows the dependence of the dust complex on the "amplitude" of the temperature fluctuations, parametrized by the width of the Gauss distribution σ. The most remarkable features are:

i) The nucleation regime (panel 2) expands remarkably with growing σ.
ii) The dust formation starts earlier, i.e. closer to the central star with increasing σ.

Fig. C shows the dependence on the correlation time λ, it is obvious that the impact of the temperature fluctuations becomes larger at the smaller correlation times.

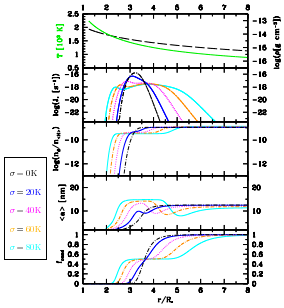

Fig. B: Influence of the parameter σ. σ = 0K corresponds to the solution of the deterministic system (Eq. 1) along the mean trajectory. $\lambda = 10^3$s for all plots. The panels show (from top to bottom): Top: mean temperature and mean density, panel 2: nucleation rate J, panel 3: dust particle density (per hydrogen core), panel 4: mean particle radius, panel 5: degree of condensation, i.e. the amount of condensable material actually condensed.

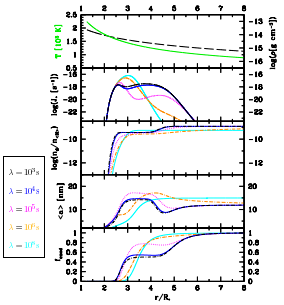

Fig. C: Influence of the correlation time λ. The parameter $\sigma = 60$K for all plots. The value $\lambda = 10^8$s gives a correlation length of about 10^{14}cm. In this case, the result corresponds to that of an averaging procedure over a set of shifted atmospheres rather than to the impact of a set of fluctuations on a single atmosphere. Accordingly the plot looks very much like a deterministic wind structure.

Outlook

The main result of the pioneering work from Dirks (2000) is that stochastical temperature fluctuations do indeed facilitate the formation of astrophysical dust particles. Since the method is not restricted a priori neither to spherical symmetry nor to a constant wind velocity (indeed one example of a wind with a velocity structure $v(r)$ is given in Dirks, 2000), we currently work on the extension of the models to more complex situations. An application of this method to other astrophysical objects (e.g. pulsating AGB-stars, Wolf-Rayet-stars) seems a promising next step.
Furthermore a series of theoretical questions has emerged: Is the assumption of a Gaussian temperature fluctuation realistic, or could other fluctuation spectra be implemented? Is the assumption of adiabatic coupling of the density good, or should other mechanisms be investigated (isobaric coupling or more sophisticated cooling functions)? By neglecting the radiative transfer, we implicitly assume an LTE situation, which introduces errors, since especially the grain evaporation is coupled to the radiation temperature rather than to the local gas temperature.

Acknowledgement:

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